An Analytical Solution to the One-Dimensional Heat Conduction–Convection Equation in Soil

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Soil heat transfer occurs by conduction and convection. Soil temperatures below infiltrating water can provide a signal for water flux. In earlier work, analysis of field measurements with a sine wave model indicated that convection heat transfer made significant contributions to the subsurface temperature oscillations. In this work, we used a Fourier series to describe soil surface temperature variations with time. The conduction and convection heat transfer equation with a multi-sinusoidal wave boundary condition was solved analytically using a Fourier transformation. Soil temperature values calculated by the single sine wave model and by the Fourier series model were compared with field soil temperature values measured at depths of 0.1 and 0.3 m below an infiltrating ponded surface. The Fourier series model provided better estimates of observed field temperatures than the sine wave model. The new model provides a general way to describe soil temperature under an infiltrating water source.

Soil heat transfer and soil water transfer occur in combination, and efforts have been made to solve soil heat and water transfer equations. Although most of the solutions use numerical techniques (e.g., Jaynes, 1990; Horton and Chung, 1991; Nassar and Horton, 1992a, 1992b), a few analytical solutions are available (Shao et al., 1998; Gao et al., 2003, 2008). Analytical solutions provide reference standards for validation of numerical solutions, and in cases where simple initial and boundary conditions occur, analytical solutions can be used to analyze natural soil thermal transfer processes (Shao et al., 1998). Gao et al. (2003, 2008) presented the following conduction– convection equation for one-dimensional, unsteady soil heat transfer in the presence of steady water flow:

$$\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial z^2}+W\frac{\partial T}{\partial z} \qquad [1]$$

where T (K) is temperature, t (h) is time, and $k \equiv \lambda/C_g$ (m^2 h^{-1}) is the soil thermal diffusivity, where λ is the soil thermal conductivity, and C_g (J K^{-1} m^{-3}) is the volumetric heat capacity of the soil; $W \equiv \partial k/\partial z - (C_w/C_g)w\theta$ (m h^{-1}), where $\partial k/\partial z$ is the vertical gradient of soil thermal diffusivity, $-(C_w/C_g)w\theta$ is the water flux density term, w (m h^{-1}) is the liquid flow rate (positive downward), θ is the volumetric water content of the soil, and C_w (J K^{-1} m^{-3}) is the heat capacity of water. These four quantities (i.e., C_g, C_w, w, and θ) are assumed to be independent of depth, z (m), for a thin soil layer. With the assumption that W is independent of time, Gao et al. (2003, 2008) obtained an analytical solution to Eq. [1].

Jaynes (1990) reported that shallow, ponded water exposed to diurnal temperature variations showed diurnal variations in soil water infiltration. Shao et al.

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(1998) developed a sine wave model with the assumption that temperature-induced viscosity changes of the ponded water led to variations in infiltration flux. Therefore, in Eq. [1], w can describe diurnal variations by assuming that $w = a_1 + a_2\sin(\omega t)$, where a_1 (m h^{-1}) and a_2 (m h^{-1}) are constants and ω (rad h^{-1}) is the angular velocity of the Earth's rotation, resulting in

$$W=\left(\frac{\partial k}{\partial z}-\frac{C_w}{C_g}a_1\theta\right)-\frac{C_w}{C_g}a_2\theta\sin(\omega t) \quad [2]$$

Assuming that $\partial k/\partial z$ is constant for a thin soil layer, $W = a + b\sin(\omega t)$, where a and b (m h^{-1}) are constants, with $a \equiv \partial k/\partial z - (C_w/C_g)a_1\theta$ and $b \equiv (C_w/C_g)a_2\theta$. Equation [1] therefore becomes

$$\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial z^2}+(a+b\sin\omega t)\frac{\partial T}{\partial z} \quad [3]$$

Shao et al. (1998) previously derived Eq. [3], although they gave a different physical explanation for a and b. If $\partial k/\partial z = 0$, however, the expressions of a and b here are the same as those of Shao et al. (1998). They applied the following initial and boundary conditions to Eq. [3]:

$$T(z,0)=f(z) \quad [4]$$

$$T(\infty,t)=T_1 \quad [5]$$

$$T(0,t)=T_0+A\sin(\omega t+\Phi) \quad [6]$$

where $f(z)$ is the initial temperature distribution in the soil profile, T_1 is defined as a constant temperature at infinite depth but is usually approximated by the temperature at a relatively large depth, T_0 is the time-average temperature of the soil surface, A is the amplitude of surface temperature oscillations, and Φ is an initial phase angle (rad). For these conditions, Shao et al. (1998) presented an analytical solution to Eq. [3].

In reality, the diurnal change in soil surface temperature does not strictly follow a single sinusoidal curve. Errors due to the assumption of a single sinusoidal temperature wave at the soil surface can be reduced by using a Fourier series to accurately describe the diurnal variation in surface soil temperature (van Wijk and de Vries, 1963). Fourier series upper boundary conditions have been used with the one-dimensional heat conduction equation to predict soil temperature, and reasonable results have been obtained (Horton et al., 1983; Heusinkveld et al., 2004; Wang et al., 2010). Therefore, in this study, for the one-dimensional heat conduction-convection equation we used the following Fourier series instead of Eq. [6] to describe surface temperature variations:

$$T(0,t)=T_0+\sum_{j=1}^{n}A_j\sin(j\omega t+\Phi_j) \qquad j=1,\,2,\,3,\,\ldots,\,n \quad [6']$$

where n is number of harmonics. When $n = 1$, Eq. [6′] is identical to Eq. [6], and Eq. [6] with Eq. [3–5] are identical to the equations used by Shao et al. (1998).

The objectives of this study were (i) to analytically solve Eq. [3] with the initial condition (Eq. [4]) and the general Fourier series surface temperature boundary condition (Eq. [6′]) and (ii) to compare field-measured soil temperature values with those calculated with analytical solutions from the surface sine wave model (Eq. [6]) and the Fourier series model (Eq. [6′]).

ANALYTICAL SOLUTION

Transformation to a Classical Heat Equation

To obtain a homogeneous boundary condition, we apply the transformation $T^* = T(z,t) - T_1$ to Eq. [3–5] and Eq. [6′], which become

$$\begin{cases}\dfrac{\partial T^*}{\partial t}=k\dfrac{\partial^2 T^*}{\partial z^2}+(a+b\sin\omega t)\dfrac{\partial T^*}{\partial z}\\ T^*(0,t)=(T_0-T_1)+\displaystyle\sum_{j=1}^{n}A_j(\sin j\omega t+\Phi_j) \qquad j=1,2,\ldots,n\\ T^*(\infty,t)=0\\ T^*(z,0)=f(z)-T_1\end{cases} \quad [7]$$

The term $a(\partial T^*/\partial z)$ then needs to be eliminated from Eq. [7]. This can be done by substituting $T^* = U(z,t)\exp(-a^2t/4k - az/2k)$ into Eq. [7], which becomes

$$\begin{cases}\dfrac{\partial U}{\partial t}=k\dfrac{\partial^2 U}{\partial z^2}-\left[\dfrac{ab}{2k}\sin(\omega t)\right]U(z,t)+b\sin(\omega t)\dfrac{\partial U}{\partial z}\\ U(0,t)=\exp\left(\dfrac{a^2t}{4k}\right)\left[(T_0-T_1)+\displaystyle\sum_{j=1}^{n}A_j\sin(j\omega t+\Phi_j)\right],\ j=1,\,2,\,\ldots,\,n\\ U(\infty,t)=0\\ U(z,0)=\left[f(z)-T_1\right]\exp\left(\dfrac{az}{2k}\right)\end{cases} \quad [8]$$

To remove the term $b\sin\omega t(\partial U/\partial z)$ from Eq. [8], we introduce a parameter $p_1(t)$, which has the dimension of length:

$$p_1(t)=\frac{b}{\omega}\left[1-(\cos\omega t)\right] \quad [9]$$

If $Z = z + p_1(t)$, then for $U(z,t)$ of Eq. [8], we have $U(z,t) = U[Z - p_1(t),t] = V(Z,t)$. The differential relationships with respect to time and depth between U and V are given by

$$\begin{cases}\dfrac{\partial U}{\partial t}=\dfrac{\partial V}{\partial t}+b\sin(\omega t)\dfrac{\partial V}{\partial Z}\\ \dfrac{\partial U}{\partial z}=\dfrac{\partial V}{\partial Z}\\ \dfrac{\partial^2 U}{\partial z^2}=\dfrac{\partial^2 V}{\partial Z^2}\end{cases} \quad [10]$$

Combining Eq. [10] with Eq. [8], we obtain

$$\begin{cases}\dfrac{\partial V}{\partial t}=k\dfrac{\partial^2 V}{\partial Z^2}-\left[\dfrac{ab}{2k}\sin(\omega t)\right]V(Z,t)\\ V\left[p_1(t),t\right]=\exp\left(\dfrac{a^2t}{4k}\right)\left[(T_0-T_1)+\displaystyle\sum_{j=1}^{n}A_j\sin(j\omega t+\Phi_j)\right] \qquad j=1,2,\ldots n\\ V(\infty,t)=0\\ V(Z,0)=\left[f(Z)-T_1\right]\exp\left(\dfrac{aZ}{2k}\right)\end{cases} \quad [11]$$

Analytical Solution of Equation [11]

The analytical solution of Eq. [11] may be found by using the Fourier sine transformation, given by

$$V(p,t)=\int_0^{\infty} V(Z,t)\sin(pZ)\mathrm{d}Z \qquad [12]$$

where p is a parameter of the Fourier transformation. By using this transformation, the problem becomes the following initial value problem of an ordinary differential equation:

$$\begin{cases} \dfrac{\mathrm{d}V(p,t)}{\mathrm{d}t}=-\left[kp^2+\dfrac{ab}{2k}\sin(\omega t)\right]V(p,t)+kp\exp\left(\dfrac{a^2t}{4k}\right) \\ \qquad \times\left[(T_0-T_1)+\displaystyle\sum_{j=1}^{n}A_j\sin(j\omega t+\Phi_j)\right] \\ V(p,0)=\displaystyle\int_0^{\infty}\left[f(Z)-T_1\right]\exp\left(\dfrac{aZ}{2k}\right)\sin(pZ)\mathrm{d}Z \end{cases} \qquad [13]$$

To solve Eq. [13], we first solve the homogeneous equation by using the method of separation of variables, and the explicit analytical solution is expressed as

$$V=\exp\left[-kp^2t+b_3\cos(\omega t)\right](V_1+V_2+V_3+V_4) \qquad [14]$$

where

$$V_1=b_1\exp(b_2t)\sum_{j=1}^{n}\left[\frac{b_2A_j\sin(j\omega t+\Phi_j)}{b_2^2+(j\omega)^2}-\frac{j\omega A_j\cos(j\omega t+\Phi_j)}{b_2^2+(j\omega)^2}\right] \qquad [15]$$

$$V_2=-b_1b_3\sum_{j=1}^{n}\left[\cos\Phi_jV_2'(j)+\sin\Phi_jV_2''(j)\right] \qquad [16]$$

where

$$\begin{aligned} V_2'(j)=&\frac{1}{2}\frac{\exp(b_2t)}{b_2^2+(j+1)^2\omega^2} \\ &\times\left\{b_2A_j\sin\left[(j+1)\omega t\right]-(j+1)\omega A_j\cos\left[(j+1)\omega t\right]\right\} \\ &+\frac{1}{2}\frac{\exp(b_2t)}{b_2^2+(j-1)^2\omega^2} \\ &\times\left\{b_2A_i\sin\left[(j-1)\omega t\right]-(j-1)\omega A_j\cos\left[(j-1)\omega t\right]\right\} \end{aligned} \qquad [17]$$

$$\begin{aligned} V_2''(j)=&\frac{1}{2}\frac{\exp(b_2t)}{b_2^2+(j+1)^2\omega^2} \\ &\times\left\{b_2A_j\cos\left[(j+1)\omega t\right]-(j+1)\omega A_j\sin\left[(j+1)\omega t\right]\right\} \\ &+\frac{1}{2}\frac{\exp(b_2t)}{b_2^2+(j-1)^2\omega^2} \\ &\times\left\{b_2A_j\cos\left[(j-1)\omega t\right]-(j-1)\omega A_j\sin\left[(j-1)\omega t\right]\right\} \end{aligned} \qquad [18]$$

$$V_3=\frac{b_4}{b_2}\exp(b_2t) \qquad [19]$$

and

$$V_4=-b_4b_3\exp(b_2\mathrm{t})\frac{b_2\cos(\omega t)+\omega\sin(\omega t)}{b_2^2+\omega^2} \qquad [20]$$

When $t = 0$, $V(p,0) = \exp(ab/2k\omega)[V_1(p,0) + V_2(p,0) + V_3(p,0) + V_4(p,0) + c]$. Therefore, $c = \exp(-ab/2k\omega)V(p,0) - [V_1(p,0) + V_2(p,0) + V_3(p,0) + V_4(p,0)]$, where $V(p,0) = \int_0^{\infty}[f(z) - T_1] \times\exp(aZ/2k)\sin(pZ)\mathrm{d}Z$.

To obtain $V(p,0)$, we assume

$$f(z)=T_1+B\exp(-qz) \qquad [21]$$

where B and q are constant coefficients. Finally, we obtain

$$V(p,0)=\frac{Bp}{(q+a/2k)^2+p^2} \qquad [22]$$

Then, based on the inverse Fourier transformation, we obtain

$$V(Z,t)=\frac{2}{\pi}\int_0^{\infty}V(p,t)\sin(pZ)\mathrm{d}p \qquad [23]$$

Analytical Solution to the Original Problem

We can now obtain the solution to the original problem (i.e., Eq. [3–5] and [6′]). From Eq. [23], we have

$$U(z,t)=\frac{2}{\pi}\int_0^{\infty}V(p,t)\sin\left[p(x)-p_1(t)\right]\mathrm{d}p \qquad [24]$$

Then $T^*(z,t)$ is given by

$$T^*(z,t)=U(z,t)\exp\left(\frac{az}{2k}-\frac{a^2t}{4k}\right) \qquad [25]$$

The solution to the original problem, Eq. [3], is given by

$$T(z,t)=T_1+T^*(z,t) \qquad [26]$$

where T_1 and $T^*(z,t)$ are given by Eq. [5] and [25], respectively. Because Eq. [24] is explicit, the final solution (Eq. [26]) is explicit rather than implicit.

The details of the derivation of the Fourier series surface temperature model (Eq. [3–5] and [6′]) are presented in the appendix. To evaluate the single sine wave model results and the Fourier series model, we used the field data collected by Jaynes (1990) and reported by Shao et al. (1998).

Field Experiments

Jaynes (1990) provided details on the instruments and the various data processing techniques used in the field experiments. The field data were collected near Phoenix, AZ. The soil was an Avondale clay loam (a fine-loamy, mixed, superactive, calcareous, hyperthermic Typic Torrifluvent). A leaching-basin method was used to measure the infiltration rate during the experiment. A 6.1- by 6.1-m area was isolated by driving a 0.4-m-wide sheet metal strip 0.2 m into the ground. The center 3.66 by 3.66 m was divided into four subbasins, 1.83 m on each side, with similar metal borders. Soil temperatures were measured hourly with Cu-constantan thermocouples at depths of 0.0, 0.05, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, and 0.6 m. Infiltration rates were measured by flow meters and corrected for changes in measured ponding depth. All of the measurements were continuous for a 120-h period, and they represented typical Arizona springtime conditions.

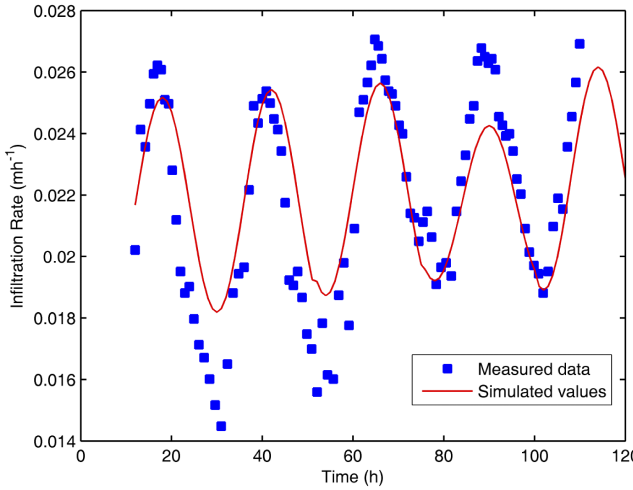

Fig. 1. Soil water infiltration rate with time (from Shao et al., 1998).

Table 1. Model parameter values.

Parameter	Value
Heat capacity of liquid water (C_w), J K^{-1} m^{-3}	4.18×10^6
Saturated soil heat capacity (C_g), J K^{-1} m^{-3}	3.14×10^6
Soil thermal diffusivity (k), m^2 h^{-1}	0.0016
Angular frequency (ω), rad h^{-1}	0.2618
Saturated hydraulic conductivity (K_s), m h^{-1}	0.024
Coefficient of linear function V_0	0.46
Coefficient of linear function V_1, K^{-1}	0.02

RESULTS AND DISCUSSION

Initial Temperature, Parameters, and Surface Infiltration

The observed initial soil temperature profile can be approximated by an exponential function (see Eq. [21] and Shao et al., 1998, Fig. 1). The initial temperature is well represented by $f(z) = 291.18\text{ K} + 12.3\text{ K} \exp(-19.07z)$.

Model parameter values are presented in Table 1. Equations [2–3] require parameters that specify the infiltration rate and the soil thermal diffusivity. In this study, we used the same function as Shao et al. (1998) to express the soil water infiltration rate w for a nearly saturated soil. Two values, V_0 and V_1, obtained by a linear regression presented in Shao et al. (1998), were used to estimate w. Based on the single sine wave model, the infiltration rate with time for this 5-d period was estimated (Fig. 1).

Diurnal Variations in Soil Temperature

Measured and modeled soil surface temperatures are shown in Fig. 2. Shao et al. (1998) modeled daily surface temperature during the 5-d period with a single sine wave function having amplitudes varying from day to day. In this study, daily surface temperature was described with a Fourier series model containing six harmonics, with amplitudes varying from day to day. The Fourier series representation of surface temperature agreed well with the measured values. Physically, the soil surface temperature is influenced by solar radiation, wind speed, and atmospheric stratification stability, and the diurnal variations in the soil surface temperature often cannot be described well by a single sine function. Mathematically, a summation of multi-harmonics agrees with measurements better than does a single sine function not only on clear-sky days but especially for characterizing multiple peaks that can occur in the diurnal variations of soil temperature on partly cloudy days. The six-harmonic Fourier series accurately captured the surface temperature dynamics. For each day, the values of mean temperature, amplitude, and phase angle obtained by fitting measured temperatures with these two models are presented in Tables 2 and 3. The single sine wave model parameters were determined with the approach of Shao et al. (1998), who determined the daily amplitude, A, as equal to half the difference between the daily maximum (T_{max}) and the daily minimum (T_{min}) surface temperature values. The daily mean temperature was determined as the daily maximum temperature minus the amplitude, $T_0 = T_{max} - A$. Once A and T_0 were known, the daily phase constant, ϕ, was determined by fitting the sine wave model to the measured temperature values.

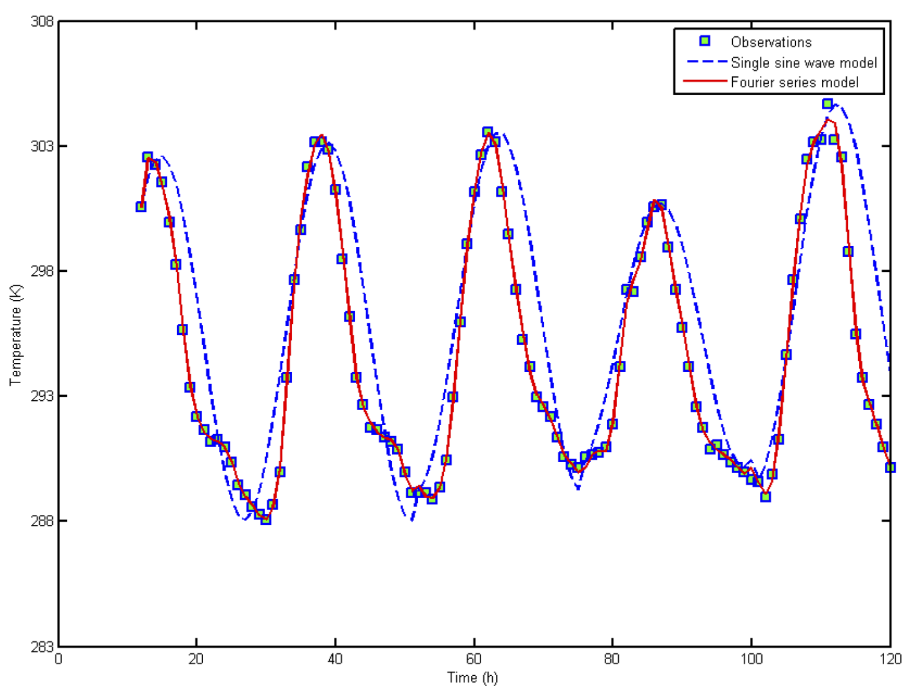

Fig. 2. Measured soil surface temperature with time, fitted single sine wave model values, and fitted Fourier series model values.

Figure 3a shows (i) temporal variations of the soil temperature measured at the 0.1-m depth, (ii) the single sine wave model calculations of soil temperature (Shao et al., 1998) at the 0.1-m depth, and (iii) the Fourier series soil temperature model (Eq. [24–26] at the 0.1-m depth. Figure 3b presents these same values of soil temperature for a depth of 0.3 m. Overall, the Fourier series soil temperature model calculated realistic soil temperatures for both 0.1 and 0.3 m. Figure 4 compares the modeled soil temperatures at depths of 0.1 and 0.3 m with the measured soil temperature values. As depth increased, the scatter in the points increased for the new analytical solution because the field soil profile was not perfectly homogeneous.

Two objective quantitative measures: root mean square error (RMSE) and normalized standard error of the estimates (NSEE) (Willmott et al., 1985) were used to estimate the prediction accuracy. The results in Table 4 indicate that the Fourier series model had lower RMSE and NSEE than the single sine wave model, with the RMSE decreasing from 1.84 to 0.96 K at the 0.1-m depth and from 1.13 to 0.93 K at the 0.3-m depth. The improved estimates of subsurface temperature can be attributed to the improved description of the surface boundary condition.

Table 2. Fourier series model values of amplitude A, phase angle f, and mean temperature T0 for daily surface temperatures.

Harmonic (n)	Day 1 T_0 = 293.45		Day 2 T_0 = 294.46		Day 3 T_0 = 294.70		Day 4 T_0 = 294.00		Day 5 T_0 = 295.20	
	A	ϕ	A	ϕ	A	ϕ	A	ϕ	A	ϕ
1	6.13	3.94	6.59	10.37	6.50	3.96	5.06	4.17	7.41	4.04
2	2.82	0.69	2.75	7.16	2.34	0.78	1.40	0.65	2.12	0.53
3	0.64	3.40	0.24	4.27	0.33	3.88	0.08	2.03	0.29	−0.09
4	0.36	1.39	0.50	3.01	0.18	0.19	0.22	3.60	0.68	3.31
5	0.27	4.23	0.34	0.88	0.05	1.18	0.39	−4.99	0.19	4.80
6	0.20	1.06	0.20	−0.43	0.14	3.76	0.22	4.78	0.13	0.31

Table 3. Single sine wave model values of amplitude A, phase angle ϕ, and mean temperature T_0 for daily surface temperatures.

Day 1 T_0 = 295.33		Day 2 T_0 = 295.59		Day 3 T_0 = 296.38		Day 4 T_0 = 295.44		Day 5 T_0 = 297.11	
A	ϕ	A	ϕ	A	ϕ	A	ϕ	A	ϕ
7.28	0.84	7.55	1.26	7.20	−2.50	5.26	−2.30	7.56	−2.70

Although we had access to only one full data set with which to compare the single sine wave and Fourier series models, the results demonstrate the improvement using the Fourier series model over the single sine wave model. The flexibility of the Fourier series model enables it to be applicable to a wide range of soil conditions. The Fourier series model is a continuum model, so it assumes continuum properties and processes. As long as field soil conditions approximate these conditions, the model should perform well in describing soil temperature distributions. If pore-scale processes, such as preferential flow, dominate the soil processes, however, the continuum assumption is violated and the model may not describe well the spatial and temporal variations of conduction and convection heat transfer in the soil.

CONCLUSIONS

The single sine wave model presented by Shao et al. (1998) describing soil temperature beneath an infiltrating water source has been expanded by changing the surface boundary temperature condition from a single sine wave to a multiple sine wave (Fourier) series. The analytical solution for the surface Fourier series condition was obtained using variable substitutions and a Fourier transformation. Subsurface soil temperatures calculated by the single sine wave model of Shao et al. (1998) and by the expanded Fourier series model were compared with field-measured soil temperature values. The Fourier series solution better matched the measured subsurface temperature than did the single sine wave model. The Fourier series analytical solution of the conduction–convection equation is straightforward and should be useful for comparison with numerical solutions of heat conduction–convection through the soil. The analytical solution is also useful for describing temperature distributions under simple, ponded surface conditions.

APPENDIX

To solve the set of Eq. [13], we first solve the homogeneous equation by using the method of separation of variables:

$$\frac{\mathrm{d}V(p,t)}{\mathrm{d}t}=-\left[kp^2+\frac{ab}{2k}\sin(\omega t)\right]V(p,t) \qquad \text{[A1]}$$

The solution is

$$\begin{aligned} V(p,t)&=c\exp\left\{\int-\left[kp^2+\frac{ab}{2k}\sin(\omega t)\right]\mathrm{d}t\right\} \\ &=c\exp\left[-kp^2t+\frac{ab}{2k\omega}\cos(\omega t)\right] \end{aligned} \qquad \text{[A2]}$$

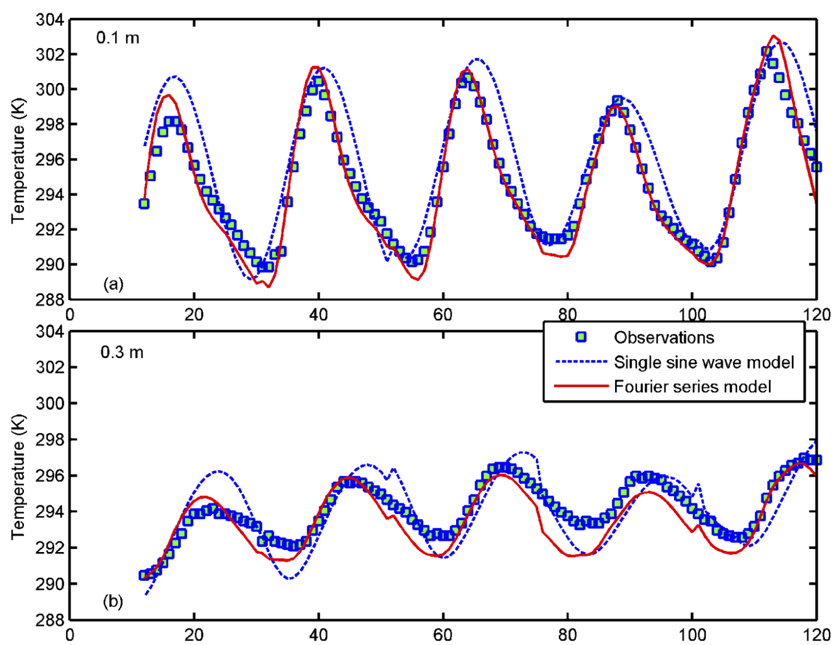

Fig. 3. Comparison of field measured soil temperatures at the (a) 0.1-m depth and (b) 0.3-m depth; temperatures calculated by a single sine wave model (Shao et al., 1998) and by a Fourier series model.

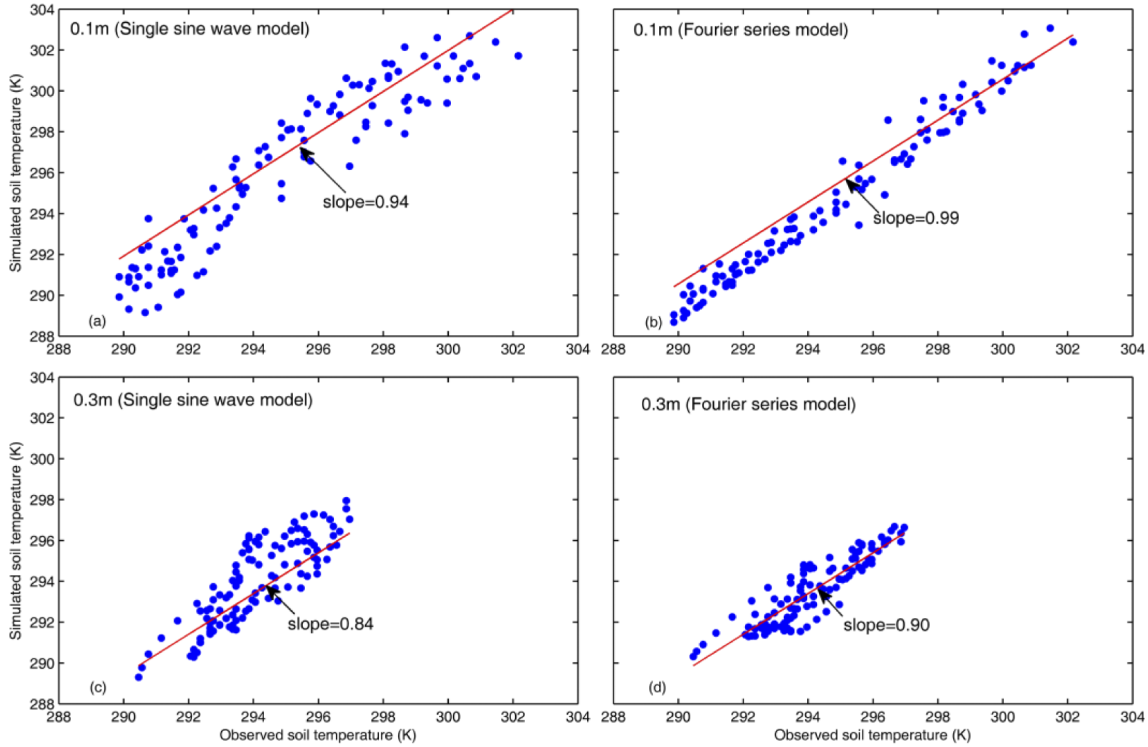

Fig. 4. Comparison of soil temperature values calculated by (a, c) a single sine wave model (Shao et al., 1998) or (b, d) a Fourier series model vs. field-measured soil temperatures at depths of (a, b) 0.1 m and (c, d) 0.3 m. The lines indicate differences from the 1:1 line as measures of the bias in the modeled temperatures.

Table 4. Root mean square error (RMSE) and normalized standard error of the estimates (NSEE) of soil temperature for the single sine wave and Fourier series models.

Depth	Models	RMSE	NSEE
m		K	
0.1	single sine wave model	1.84	0.0063
	Fourier series model	0.83	0.0028
0.3	single sine wave model	1.13	0.0039
	Fourier series model	0.93	0.0032

where c is a constant of integration. The method of variation of parameters is applied to Eq. [A2] for solving Eq. [13], i.e., let

$$V(p,t)=Y\exp\left[-kp^2t+\frac{ab}{2k\omega}\cos(\omega t)\right] \quad [A3]$$

where Y is a variable rather than a constant. Thus the partial differential equation of $V(p,t)$ with respect to time is

$$\frac{\partial V(p,t)}{\partial t}=\frac{\partial Y}{\partial t}\exp\left[-kp^2t+\frac{ab}{2k\omega}\cos(\omega t)\right] +Y\left[-kp^2-\frac{ab}{2k\omega}\sin(\omega t)\right] \times\exp\left[-kp^2t+\frac{ab}{2k\omega}\cos(\omega t)\right] \quad [A4]$$

Substituting Eq. [A4] into Eq. [13] leads to

$$\frac{\partial Y}{\partial t}\exp\left[-kp^2t+\frac{ab}{2k\omega}\cos(\omega t)\right]+ Y\left[-kp^2-\frac{ab}{2k\omega}\sin(\omega t)\right]\times \exp\left[-kp^2t+\frac{ab}{2k\omega}\cos(\omega t)\right]=-\left[kp^2+\frac{ab}{2k}\sin(\omega t)\right]\times Y\exp\left[-kp^2t+\frac{ab}{2k\omega}\cos(\omega t)\right]+ kp\exp\left(\frac{a^2t}{4k}\right)\left[(T_0-T_1)+\sum_{j=1}^{n}A_j\left(\sin j\omega t+\Phi_j\right)\right]$$

This can be simplified to

$$\frac{\partial Y}{\partial t}=kp\left[(T_0-T_1)+\sum_{j=1}^{n}A_j\sin(j\omega t+\Phi_j)\right]\times \exp\left[kp^2t-\frac{ab}{2k\omega}\cos(\omega t)+\frac{a^2t}{4k}\right] \quad [A5]$$

Integrating Eq. [A5], we obtain

$$Y=\int\left\{kp\left[(T_0-T_1)+\sum_{j=1}^{n}A_j\sin(j\omega t+\Phi_j)\right]\times \exp\left[kp^2t-\frac{ab}{2k\omega}\cos(\omega t)+\frac{a^2t}{4k}\right]\right\}dt+c$$
$$=kp\int\left\{\sum_{j=1}^{n}A_j\sin(j\omega t+\Phi_j)\times \exp\left[kp^2t-\frac{ab}{2k\omega}\cos(\omega t)+\frac{a^2t}{4k}\right]\right\}dt+ kp(T_0-T_1)\times \int\left\{\exp\left(kp^2t-\frac{ab}{2k\omega}\cos(\omega t)+\frac{a^2t}{4k}\right)\right\}dt+c \quad [A6]$$

Substituting Eq. [A6] into Eq. [A3] leads to

$$V\left(p,t\right)=\exp\left[-kp^2t+\frac{ab}{2k\omega}\cos\left(\omega t\right)\right]\left(M+J+c\right) \qquad \text{[A7]}$$

where

$$M=kp\left(T_0-T_1\right)\int\exp\left[\frac{a^2}{4k}t+kp^2t-\frac{ab}{2k\omega}\cos\left(\omega t\right)\right]\mathrm{d}t \qquad \text{[A8]}$$

and

$$\begin{aligned}J=kp&\int\sum_{j=1}^{n}A_j\sin\left(j\omega t+\Phi_j\right)\\&\times\exp\left[\frac{a^2}{4k}t+kp^2t-\frac{ab}{2k\omega}\cos\left(\omega t\right)\right]\mathrm{d}t\end{aligned} \qquad \text{[A9]}$$

To complete the solution of Eq. [A7], we have to determine M and J. When $(ab/2k\omega)\cos(\omega t) << 1$, the following approximate equation can be applied to Eq. [A8–A9]:

$$\exp\left[-\frac{ab}{2k\omega}\cos\left(\omega t\right)\right]\approx 1-\frac{ab}{2k\omega}\cos\left(\omega t\right) \qquad \text{[A10]}$$

We let

$$\begin{cases}b_1=kp\\ b_2=\dfrac{a^2}{4k}+kp^2\\ b_3=\dfrac{ab}{2k\omega}\\ b_4=kp\left(T_0-T_1\right)\end{cases} \qquad \text{[A11]}$$

then

$$\begin{aligned}M&\approx kp\left(T_0-T_1\right)\int\exp\left(\frac{a^2}{4k}t+kp^2t\right)\left[1-\frac{ab}{2k\omega}\cos\left(\omega t\right)\right]\mathrm{d}t\\&=b_4\int\exp\left(b_2t\right)\mathrm{d}t-b_4b_3\int\cos\left(\omega t\right)\exp\left(b_2t\right)\mathrm{d}t\\&=\frac{b_4}{b_2}\exp\left(b_2t\right)-b_4b_3\int\cos\left(\omega t\right)\exp\left(b_2t\right)\mathrm{d}t\end{aligned} \qquad \text{[A12]}$$

Then let

$$\begin{aligned}N&=-b_4b_3\int\cos\left(\omega t\right)\exp\left(b_2t\right)\mathrm{d}t\\&=-b_4b_3\left[\frac{\cos\left(\omega t\right)}{b_2}\exp\left(b_2\mathrm{t}\right)-\int\left(-\omega\right)\sin(\omega t)\exp\left(\frac{a^2}{4k}+kp^2\right)\mathrm{d}t\right]\\&=-b_4b_3\frac{\cos\left(\omega t\right)}{b_2}\exp\left(b_2t\right)-b_4b_3\omega\int\sin(\omega t)\exp\left(b_2t\right)\mathrm{d}t\\&=-b_4b_3\frac{\cos\left(\omega t\right)}{b_2}\exp\left(b_2\mathrm{t}\right)-b_4b_3\omega\frac{1}{b_2}\left[\frac{\sin\left(\omega t\right)}{b_2}\right]\exp\left(b_2\mathrm{t}\right)\\&\quad-\frac{\omega}{b_2}\int\cos\left(\omega t\right)\exp\left(b_2t\right)\mathrm{d}t\\&=-b_4b_3\frac{\cos\left(\omega t\right)}{b_2}\exp\left(b_2\mathrm{t}\right)-b_4b_3\omega\frac{\sin\left(\omega t\right)}{{b_2}^2}\exp\left(b_2\mathrm{t}\right)\\&\quad-b_4b_3\frac{\omega^2}{{b_2}^2}\int\cos\left(\omega t\right)\exp\left(b_2t\right)\mathrm{d}t\end{aligned}$$

Simplifying, we have

$$\begin{aligned}N&=-b_4b_3\exp\left(b_2\mathrm{t}\right)\frac{\left[b_2\cos\left(\omega t\right)+\omega\sin\left(\omega t\right)\right]/b_2^2}{\left(b_2^2+\omega^2\right)/b_2^2}\\&=-b_4b_3\exp\left(b_2\mathrm{t}\right)\frac{b_2\cos\left(\omega t\right)+\omega\sin\left(\omega t\right)}{b_2^2+\omega^2}\end{aligned} \qquad \text{[A13]}$$

Substituting Eq. [A13] into Eq. [A12], we obtain

$$\begin{aligned}M=&\frac{b_4}{b_2}\exp\left(b_2t\right)\\&-b_4b_3\exp\left(b_2\mathrm{t}\right)\frac{b_2\cos\left(\omega t\right)+\omega\sin\left(\omega t\right)}{b_2^2+\omega^2}\end{aligned} \qquad \text{[A14]}$$

In the same way,

$$\begin{aligned}J&=kp\int\sum_{j=1}^{n}A_j\sin\left(j\omega t+\Phi_j\right)\times\\&\qquad\exp\left[\frac{a^2}{4k}t+kp^2t-\frac{ab}{2k\omega}\cos\left(\omega t\right)\right]\mathrm{d}t\\&\approx kp\int\sum_{j=1}^{n}A_j\sin\left(j\omega t+\Phi_j\right)\times\\&\qquad\exp\left(\frac{a^2}{4k}t+kp^2t\right)\left(1-\frac{ab}{2k\omega}\cos\omega t\right)\mathrm{d}t\\&=b_1\int\sum_{j=1}^{n}A_j\sin\left(j\omega t+\Phi_j\right)\exp\left(b_2t\right)\mathrm{d}t\\&\quad-b_1\int b_3\cos\left(\omega t\right)\sum_{j=1}^{n}A_j\sin\left(j\omega t+\Phi_j\right)\exp\left(b_2t\right)\mathrm{d}t\\&=b_1J_1-b_1b_3J_2\end{aligned} \qquad \text{[A15]}$$

where

$$\begin{aligned}J_1&=\int\sum_{j=1}^{n}A_j\sin\left(j\omega t+\Phi_j\right)\exp\left(b_2t\right)\mathrm{d}t\\&=\sum_{j=1}^{n}\int A_j\sin\left(j\omega t+\Phi_j\right)\exp\left(b_2t\right)\mathrm{d}t\\&=\sum_{j=1}^{n}J_1\left(j\right)\end{aligned} \qquad \text{[A16]}$$

where

$$\begin{aligned}J_1(j)&=\int A_j\sin\left(j\omega t+\Phi_j\right)\exp\left(b_2t\right)\mathrm{d}t\\&=A_j\left[\frac{\sin\left(j\omega t+\Phi_j\right)}{b_2}\exp\left(b_2t\right)-\frac{j\omega}{b_2}\int\cos\left(j\omega t+\Phi_j\right)\exp\left(b_2t\right)\mathrm{d}t\right]\\&=A_j\left[\begin{aligned}&\frac{\sin\left(j\omega t+\Phi_j\right)}{b_2}\exp\left(b_2t\right)-\frac{j\omega\cos\left(j\omega t+\Phi_j\right)}{{b_2}^2}\times\\&\qquad\exp\left(b_2t\right)+\frac{j\omega}{b_2^2}\int\left(-j\omega\right)\sin\left(j\omega t+\Phi_j\right)\exp\left(b_2t\right)\mathrm{d}t\end{aligned}\right]\\&=A_j\left[\frac{\sin\left(j\omega t+\Phi_j\right)}{b_2}\exp\left(b_2t\right)-\frac{j\omega\cos\left(j\omega t+\Phi_j\right)}{b_2^2}\exp\left(b_2t\right)\right]-\frac{\left(j\omega\right)^2}{b_2^2}J_2\left(j\right)\end{aligned}$$

so that

$$J_1\left(j\right)=\exp\left(b_2t\right)\left[\frac{b_2A_j\sin\left(j\omega t+\Phi_j\right)}{b_2^2+\left(j\omega\right)^2}-\frac{j\omega A_j\cos\left(j\omega t+\Phi_j\right)}{b_2^2+\left(j\omega\right)^2}\right] \qquad \text{[A17]}$$

Substituting Eq. [A17] into Eq. [A16] results in

$$\begin{aligned}J_1=&\exp\left(b_2t\right)\\&\times\sum_{j=1}^{n}\left[\frac{b_2A_j\left(\sin j\omega t+\Phi_i\right)}{{b_2}^2+\left(j\omega\right)^2}-\frac{j\omega A_j\cos\left(j\omega t+\Phi_j\right)}{{b_2}^2+\left(j\omega\right)^2}\right]\end{aligned} \qquad \text{[A18]}$$

In Eq. [A15],

$$
\begin{aligned}
J_2 &= \int \sum_{j=1}^{n} A_j \cos(\omega t)\sin\left(j\omega t+\Phi_j\right)\exp(b_2 t)\,\mathrm{d}t \\
&= \sum_{j=1}^{n} \int A_j \cos(\omega t)\sin\left(j\omega t+\Phi_j\right)\exp(b_2 t)\,\mathrm{d}t \\
&= \sum_{j=1}^{n} J_2(j)
\end{aligned}
\qquad \text{[A19]}
$$

where

$$
\begin{aligned}
J_2(j) &= \int A_j \cos(\omega t)\times\sin\left(j\omega t+\Phi_j\right)\exp(b_2 t)\,\mathrm{d}t \\
&= \int A_j \cos(\omega t) \\
&\quad \times\left[\sin(j\omega t)\cos\Phi_j+\cos(j\omega t)\sin\Phi_j\right] \\
&\quad \times\exp(b_2 t)\,\mathrm{d}t \\
&= \cos\Phi_j \int \exp(b_2 t) A_j \cos(\omega t)\sin(j\omega t)\,\mathrm{d}t \\
&\quad +\sin\Phi_j \int \exp(b_2 t)\cos(j\omega t)\cos(\omega t)\,\mathrm{d}t \\
&= \cos\Phi_j J_2'(j)+\sin\Phi_j J_2''(j)
\end{aligned}
\qquad \text{[A20]}
$$

Therefore,

$$
\begin{aligned}
J_2'(j) &= \int \exp(b_2 t) A_j \cos(\omega t)\sin(j\omega t)\,\mathrm{d}t \\
&= \frac{1}{2}\left\{\int \exp(b_2 t) A_j \sin\left[(j+1)\omega t\right]\mathrm{d}t\right. \\
&\quad \left. +\int \exp(b_2 t) A_j \sin\left[(j-1)\omega t\right]\mathrm{d}t\right\}
\end{aligned}
\qquad \text{[A21]}
$$

where

$$
\begin{aligned}
&\int \exp(b_2 t) A_j \sin\left[(j+1)\omega t\right]\mathrm{d}t= \\
&\frac{\exp(b_2 t)}{b_2^{\,2}+(j+1)^2\omega^2}\left\{[b_2 A_j \sin\left[(j+1)\omega t\right]\right. \\
&\left.-(j+1)\omega A_j \cos\left[(j+1)\omega t\right]\right\}
\end{aligned}
\qquad \text{[A22]}
$$

and

$$
\begin{aligned}
\int \exp(b_2 t) A_j \sin\left[(j-1)\omega t\right]\mathrm{d}t &= \frac{\exp(b_2 t)}{b_2^{\,2}+(j-1)^2\omega^2} \\
&\quad \times\left\{b_2 A_j \sin\left[(j-1)\omega t\right]-(j-1)\omega A_j \cos\left[(j-1)\omega t\right]\right\}
\end{aligned}
\qquad \text{[A23]}
$$

Substituting Eq. [A21–A22] into Eq. [A20] gives

$$
\begin{aligned}
J_2' &= \frac{1}{2}\frac{\exp(b_2 t)}{b_2^{\,2}+(j+1)^2\omega^2}\left\{b_2 A_j \sin\left[(j+1)\omega t\right]\right. \\
&\qquad \left.-(j+1)\omega A_j \cos\left[(j+1)\omega t\right]\right\} \\
&\quad +\frac{1}{2}\frac{\exp(b_2 t)}{b_2^{\,2}+(j-1)^2\omega^2}\left\{b_2 A_j \sin\left[(j-1)\omega t\right]\right. \\
&\qquad \left.-(j-1)\omega A_j \cos\left[(j-1)\omega t\right]\right\}
\end{aligned}
\qquad \text{[A24]}
$$

In Eq. [A20],

$$
\begin{aligned}
J_2''(j) &= \int \exp(b_2 t) A_j \cos(j\omega t)\cos(\omega t)\,\mathrm{d}t \\
&= \frac{1}{2}\left\{\int \exp(b_2 t) A_j \cos\left[(j+1)\omega t\right]\mathrm{d}t\right. \\
&\quad \left.+\int \exp(b_2 t) A_j \cos\left[(j-1)\omega t\right]\mathrm{d}t\right\}
\end{aligned}
\qquad \text{[A25]}
$$

where

$$
\begin{aligned}
\int \exp(b_2 t) A_j \cos\left[(j+1)\omega t\right]\mathrm{d}t &= \frac{\exp(b_2 t)}{b_2^{\,2}+(j+1)^2\omega^2}\times \\
&\quad \left\{b_2 A_j \cos\left[(j+1)\omega t\right]+(j+1)\omega A_j \sin\left[(j+1)\omega t\right]\right\}
\end{aligned}
\qquad \text{[A26]}
$$

and

$$
\begin{aligned}
&\int \exp(b_2 t) A_j \sin\left[(j-1)\omega t\right]\mathrm{d}t= \\
&\frac{\exp(b_2 t)}{b_2^{\,2}+(j-1)^2\omega^2} \\
&\times\left\{b_2 A_j \cos\left[(j-1)\omega t\right]\right. \\
&\quad \left.-(j-1)\omega A_j \sin\left[(j-1)\omega t\right]\right\}
\end{aligned}
\qquad \text{[A27]}
$$

Substituting Eq. [A26] and [A27] into Eq. [A25] yields

$$
\begin{aligned}
J_2'' &= \frac{1}{2}\frac{\exp(b_2 t)}{b_2^{\,2}+(j+1)^2\omega^2} \\
&\quad \times\left\{b_2 A_j \cos\left[(j+1)\omega t\right]\right. \\
&\qquad \left.-(j+1)\omega A_j \sin\left[(j+1)\omega t\right]\right\} \\
&\quad +\frac{1}{2}\frac{\exp(b_2 t)}{b_2^{\,2}+(j-1)^2\omega^2} \\
&\quad \times\left\{b_2 A_j \cos\left[(j-1)\omega t\right]\right. \\
&\qquad \left.-(j-1)\omega A_j \sin\left[(j-1)\omega t\right]\right\}
\end{aligned}
\qquad \text{[A28]}
$$

The value of J_2 can be therefore obtained by substituting Eq. [A24] and [A28] into Eq. [A20].

Returning to Eq. [A7],

$$
\begin{aligned}
V &= \exp\left(-kp^2 t+b_3\cos\omega t\right)(M+J+c) \\
&= \exp\left(-kp^2 t+b_3\cos\omega t\right)(V_1+V_2+V_3+V_4)
\end{aligned}
\qquad \text{[A29]}
$$

where V_1, V_2, V_3, and V_4 were defined in the main text.

As mentioned in the main text, the initial soil temperature with depth can be approximated by Eq. [21]. Then

$$
f(Z)=T_1+B\exp\left\{-q\left[Z-p_1(t=0)\right]\right\} \qquad \text{[A30]}
$$

Because $p_1(t = 0) = 0$,

$$
f(Z)=T_1+B\exp(-qZ) \qquad \text{[A31]}
$$

Therefore,

$$
\begin{aligned}
V(p,0) &= \int_0^{\infty}\left[f(Z)-T_1\right]\exp\left(\frac{aZ}{2k}\right)\sin(pZ)\,\mathrm{d}Z \\
&= \int_0^{\infty} B\exp(-qZ)\exp\left(-\frac{aZ}{2k}\right)\sin(pZ)\,\mathrm{d}Z \\
&= B\int_0^{\infty}\exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\sin(pZ)\,\mathrm{d}Z
\end{aligned}
\qquad \text{[A32]}
$$

where

$$\int_0^\infty \exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\sin(pZ)\,dZ = -\sin(pZ)\frac{1}{q+a/2k}\exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\Big|_0^\infty + \frac{p}{q+a/2k}\int_0^\infty \exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\cos(pZ)\,dZ \quad [A33]$$

$$-\sin(pZ)\frac{1}{q+a/2k}\exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\Big|_0^\infty = \lim_{Z\to\infty}\left\{-\sin(pZ)\frac{1}{q+a/2k}\exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\right\}=0 \quad [A34]$$

and

$$\frac{p}{q+a/2k}\int_0^\infty \exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\cos(pZ)\,dZ = -\frac{p}{(q+a/2k)^2}\exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\cos(pZ)\Big|_0^\infty - \frac{p^2}{(q+a/2k)^2}\times\int_0^\infty \exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\sin(pZ)\,dZ = \frac{p}{(q+a/2k)^2} - \frac{p^2}{(q+a/2k)^2}\times\int_0^\infty \exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\sin(pZ)\,dZ \quad [A35]$$

Substituting Eq. [A34–A35] into Eq. [A33] yields

$$\int_0^\infty \exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\sin(pZ)\,dZ = \frac{p}{(q+a/2k)^2} - \frac{p^2}{(q+a/2k)^2}\times\int_0^\infty \exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\sin(pZ)\,dZ$$

i.e.,

$$\int_0^\infty \exp\left[-\left(q+\frac{a}{2k}\right)Z\right]\sin(pZ)\,dZ = \frac{p}{(q+a/2k)^2+p^2}$$

Finally, we obtain Eq. [22].

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